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A Simple Rule for Generating Equivalent Models in Covariance Structure Modeling

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This study introduces the replacing rule as a simplification of Stelzl's (1986) four rules for the generation of recursive equivalent models. The replacing rule is applicable to nonrecursive as well as recursive models, and generates equivalent models through the replacement of direct paths with residual correlations, through the replacement of residual correlations with direct paths, or through the inversion of path directions. Examples of the use of the replacing rule are provided, and its advantages over Stelzl's four rules are discussed.

Increased application of covariance structure modeling (CSM) as a general method of testing structural relations among variables has been accompanied by a greater awareness of the data analytic problems associated with CSM. One problem which has eluded the attention of most researchers, save for a few (e.g., Bentler & Chou, 1987; Duncan, 1969; Stelzl, 1986) is model equivalence. Equivalent models are equivalent at a mathematical level, although they have distinct path diagrams and provide different interpretations.

If model equivalence is not seriously considered in empirical research, a best fitting model among alternative models may be interpreted as a plausible model. In *this* case, the term *optimal* model refers to *this* best fitting model. But because many distinct models can fit a given data set equally well, there is a need to distinguish the optimal model from a best fitting model. The existence of multiple, equally good-fitting models or equivalent models rules out the use of the term *best* fitting model. The term *optimal model* will be defined as the most theoretically plausible model of the equivalent models which could have generated the data. The optimal model will show a very good fit with large sample data however; there will be many implausible models showing this same degree of fit.

Equivalent models have been defined (Jöreskog & Sörbom, 1981; Stelzl, 1986) as follows. For any given model, there will exist alternative models which generate identical estimates of population covariance matrices and, as a result,

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fit the observed data equally well (Bentler, 1980; Duncan, 1969, 1975; Heise, 1975; James, Mulaik, & Brett, 1982; Jöreskog & Sörbom, 1981; Kenny, 1979). Models are called *equivalent* when they reproduce the same set of covariance matrices. Equal fit is a necessary result of model equivalence. However, equal fit is not proof of model equivalence because fit measures from two models may only appear identical due to rounding error. Luijben (1988) presents some conditions for two models of equal fit to be equivalent models.

Model equivalence has important implications for the search for the optimal model, many of which were discussed by Lee (1987). Given the existence of models equivalent to one's own, Lee argued for the desirability of identifying and evaluating the equivalent models. Identifying equivalent models will yield support for a given model if the equivalent models are ruled out or can reveal previously unrecognized plausible alternative models.

Although the possibility of equivalent models has never been rejected, few researchers have raised this issue in regard to the conduct of empirical research. Duncan (1969), in an early article, presented nine distinct models that were equally consistent with the data and emphasized that the choice among these models must go beyond data analysis. Some of the nine models in Duncan are recursive, others are non-recursive. Karlin, Cameron, and Chakraborty (1983) and Bentler and Chou (1987) pointed out that the complete reversal of a path diagram generates an equivalent models in some cases.

However, Stelzl (1986) seems to be the first to investigate model equivalence in a systematic way. Stelzl developed four rules for generating equivalent models, but her rules were restricted to recursive models. In this study, our purpose is to develop a simple rule that can incorporate the applicability of Stelzl's four rules and be applied to nonrecursive models as well. In the course of the discussion, a review of model equivalence in CSM will be presented, followed by an introduction of our rule, which has been termed the "Replacing Rule." Finally, Stelzl's rules will be compared with the replacing rule, and the relative advantages of the replacing rule will be discussed in full.

Review of Model Equivalence in CSM

Definition and Implications

Despite several different mathematical formulations of CSM (e.g., Bentler & Weeks, 1980; Jöreskog, 1974, 1977; McArdle & McDonald, 1984; McDonald, 1978), model estimates of covariance matrices and model equivalence can be generally explained. Given a sample covariance matrix S for observed variables, each nonredundant variance/covariance element in S is written as a mathematical function of the model's parameters. This equation is called a normal equation (Duncan, 1975;

James et al., 1982). Given k observed variables, there are $k(k+1)/2$ normal equations relating information from the observed data to the parameters in the model. Unknown or free parameters are estimated by solving the normal equations. Substitution of the parameter estimates into the normal equations results in population variances/covariances estimated by the model. A symbol E will be used to refer to this estimated (fitted) covariance matrix[1]. Equivalence of $\sum$s between two models defines model equivalence in CSM; models generating identical Es are identified as equivalent models regardless of their distinct causal patterns (Jöreskog & Sörbom, 1981; Stelzl, 1986).

The definition of model equivalence given by Jöreskog and Sörbom (1981) and Stelzl (1986) will be supplemented by distinguishing between the concepts of *equivalence-in-principle* and *equivalence occurring only empirically*. For a particular model, a set of equivalent models exists that generate identical Es with any input data. This type of model equivalence will be called equivalence-in-principle (EIP). Some models generate identical Es only when they are fitted to a specific data set. This type of model equivalence is sample-specific and cannot be generalized to all data sets. Thus it will be called empirical occurrence of equivalence (EOE). Application of Stelzl's rule and the replacing rule generates EIP models.

When a model has very few over-identifying restrictions and/or many of the parameter values are equal or very close, EOE may be observed (Lee, 1987). When parameter values are equal or very close, normal equations simplify and over-identifying equations are likely to become equal. As a result, the number of nonredundant normal equations may equal the number of unknown parameters. This situation is likely to occur in the numerical analysis of CSM when there are very few over-identifying restrictions in the model. An equal number of normal equations and free parameters is a necessary condition for a model to be just-identified. When two models are just-identified, their Es are exactly the same as the input data S; that is, they generate identical Es.

Regardless of EIP or EOE, the numerical value for a particular fit index defined by the discrepancy of E and S will be the same across the models. Sometimes, the value for a particular fit index may be identical across two models due only to the rounding procedure in the computer program. All the elements in the Es need not be correspondingly identical for identical fit values to occur. This case should not be called model equivalence because model equivalence requires an identical value of each element $\sum$ across the models. In this article, we refer to only EIP as *model equivalence*, unless noted otherwise.

Lee (1987) discussed implications of model equivalence at different stages of applying CSM: In modeling at the hypothesis generation stage, during the specification search, at the end of the specification search, and for the final acceptance of the

[1] The fitted covariance matrix of observed variables is given as the "fitted moment matrix" in the printed output of LISREL (Jöreskog & Sörbom, 1981).

model. During the hypothesis generation stage, alternative models may be constructed a priori such that some of these competing models are equivalent. Application of rules (e.g., Stelzl, 1986) can reveal which models are equivalent. In fact, it would be conceivable, though unlikely, that some situations exist in which all competing models of interest are equivalent in which case the collection of data becomes meaningless (Stelzl, 1986). During a specification search, equivalent models are generated when LISREL modification indices have the same value on two or more fixed/constrained parameters. The models defined by freeing any one of the corresponding parameters show equal fit. Equivalent *E*s are observed for these equally fitting models[2]. The equivalent models observed in this way may represent EIP or EOE.

At the end of a specification search, it is conceivable that one may have selected a model equivalent to the optimal model rather than the optimal model itself. Regardless of the stage the equivalent models are generated, empirical data or empirical fit indices cannot be used to decide among the equivalent alternatives. Instead, this decision can only be accomplished with substantive theory because equivalent models have distinct path diagrams, each of which represents a different causal hypothesis.

Considering the problematic occurrence of model equivalence in empirical research, there is a need for a priori rules to test model equivalence at the hypothesis generating stage and/or to generate equivalent models to a model finally chosen through specification searches. Stelzl (1986) developed four a priori rules to generate equivalent models[3]. Thus, we will now discuss Stelzl's approach to model equivalence, followed by a discussion of our own approach. It will be seen that one of the differences between the two approaches lies in the nature of the restrictions placed on the generation of equivalent models.

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[2] Luijben (1988) approaches model equivalence in the light of modification indices (MIs). He describes a sufficient condition in which model modification based on identical MIs leads to equivalent models. However, it is not empirically known how often model modifications based on identical MIs do not satisfy his sufficient conditions; it is not known how often modified models based on identical MIs may not produce identical *E*s, that is, equivalent models. Authors have always observed identical *E*s after model modifications based on identical values of MIs.

[3] In an application to artificial intelligence, Glymour, Scheines, Kelly, and Spirtes (1987) developed a computer program (TETRAD) to find candidate models which would fit at least as well as a given model. TETRAD provides these candidate models without fitting the models to the data. This program may be useful in empirically finding equivalent models. Many different causal patterns suggested by TETRAD may generate identical *E*s when they are actually fitted to data. However, our interest here is in a priori rules which can determine equivalent models without using data.

Restrictions in Developing A Priori Rules

A covariance structure model consists of two submodels: A structural model specifying the relationships among latent variables, and a measurement model specifying the relationships between latent variables and measured variables. Stelzl's (1986) rules and the rules we develop here for generating equivalent models are relevant to only the structural portion of a covariance structure model. In discussing the rules for generating equivalent models, the measurement model is assumed to be known and fixed[4]. In the present study, the term *model* will refer to a structural model. There are cases where equivalent models can be obtained when the restriction of a fixed measurement model is dropped, but systematic rules have not been determined for such cases. Thus, model equivalence will be restricted to the equivalence of the structural portion of two or more covariance structure models in this study.

Although equivalent models can be generated by using equality constraints or other functional constraints, both Stelzl's (1986) rules and the rule we are developing here concern structural models with zero path coefficient as the only type of restriction. In both Stelzl's rules and our rule, the number of free parameters do not change, only the specification of fixed and free parameters changes in generating equivalent models.

Whereas Stelzl's (1986) rules require that a model be recursive, the replacing rule places a less restrictive condition on the recursiveness of the model. The replacing rule requires *limited block-recursiveness* of a model. The term *block-recursiveness* is used in the same sense that the term *block-recursive equation systems* is used in econometrics (Kmenta, 1971; Pindyck & Rubinfeld, 1981): In terms of a path diagram, a block-recursive equation system is expressed in such a way that relations among variables within each block may be recursive or nonrecursive but the relations across blocks are always recursive.

Assuming that any model can be viewed as block-recursive, a model can be divided into three blocks that are recursive across themselves: A preceding block, a focal block, and a succeeding block. In this study, the focal block refers to a set of equations or their path diagram to which the replacing rule will be applied.

Relations within the preceding block or succeeding block may be recursive or nonrecursive. However, it is essential that the relations within a focal block be recursive to apply the replacing rule. That is, criterion variables in the regression equations of a focal block should be connected either by a direct path or a residual correlation, but not by both in the path diagram. Because of this limit

[4] Working with a structural model given a fixed measurement model is the approach that has been formally defined and evaluated by Anderson and Gerbing (1988).

in the focal block, we say that the replacing rule requires *limited block-recursiveness*, not just block-recursiveness. Compared to the recursiveness of the whole model required by Stelzl's approach, this approach is less restrictive.

Replacing Rule

In a relationship $X \rightarrow Y$, we will call X the source variable and Y the effect variable; Ux and Uy stand for the residuals of X and Y. Given a focal block and the limited block-recursiveness of a model, the replacing rule is defined as follows: A direct path, that is, $X \rightarrow Y$, in the focal block can be replaced by a residual correlation between them, the correlation between Ux and Uy, as long as the predictor variables[5] of the effect variable (Y) are the same as or include those of the source variable (X). The reverse application of this procedure will result in an equivalent model as well: any residual correlation (e.g., the correlation between Ux and Uy) within a focal block where the condition of limited block-recursiveness holds in the model can be replaced by a direct path, $X \rightarrow Y$ or $X \leftarrow Y$, the choice between which is made so that the effect variable has the same or more predictors in the preceding block (PBL) than the source variable has after the change.

This rule is not affected by the recursiveness or nonrecursiveness in the preceding or succeeding block. This property of the replacing rule will now be mathematically shown. Assume an $X \rightarrow Y$ relationship within a focal block. Let X have direct paths with predictor variables ($P1, \ldots, P\text{m}$) and Y have both ($P1, \ldots, P\text{m}$) and ($Q1, \ldots, Q\text{n}$) as predictor variables in the PBL. The process of replacing the direct path from X to Y by their residual correlation in the focal block will be shown first.

The structural equations of X and Y are described as:

$$X = F(P1, \ldots, P\text{m}) + u, \tag{1}$$

$$Y = G(P1, \ldots, P\text{m}) + H(Q1, \ldots, Q\text{n}) + bX + v, \tag{2}$$

where COV(u,v) is non-zero, u is the residual of X, v is the residual of Y and b, F, G, or H represent path coefficients. The deletion of $X \rightarrow Y$ is equivalent to a reduction of Equation 2 by substituting Equation 1 into the right hand side of Equation 2. Then, variable X is removed from Equation 2 resulting in:

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[5] We use the term *predictor* in the familiar regression analysis-sense of the term. In terms of a path diagram, a predictor has a direct path to a dependent variable. In the discussion of the replacing rule, the term *predictor* will refer to the predictor of the focal block, not to the predictor in a predictor-criterion relationship between variables within a focal block.

$$Y = J(P1, \ldots, Pm) + H(Q1, \ldots, Qn) + (bu + v) \text{ or}$$

$$Y = J(P1, \ldots, Pm) + H(Q1, \ldots, Qn) + e, \tag{3}$$

where J represents the changed values of path coefficients.

$$\mathrm{COV}(u,e) = \mathrm{COV}(u,bu + v) = b\ \mathrm{VAR}(u) + 0 \neq 0, \tag{4}$$

that is, the residual correlation between X and Y becomes non-zero. Thus, in terms of model specification, deleting $X \rightarrow Y$ requires freeing the residual correlation between X and Y. No other specification is altered. This procedure does not change the number of parameters.

Replacing the direct path from X to Y with their residual correlation is nothing more than a path diagramic representation of the algebraic reduction of Equation 2. If the algebraic reduction is correctly reflected in the path diagram, the covariances among the variables are unaltered. The covariances among the variables in the PBL do not change because they are not affected by any change in the following blocks. The relationships between the PBL and focal block are unchanged as well.

The covariance between X and Y does not change by the reduction of Equation 2 into Equation 3. The covariance between X and Y can be obtained either from Equations 1 and 2 or from Equations 1 and 3[6]. Both ways are algebraically equivalent. Thus, the covariances among the variables in the PBL and focal block are unaltered. Because the specification of the succeeding block is unaltered, the covariances of the variables in the whole model are equivalent to those before applying the replacing rule. This equivalence necessitates the equivalence of Es between the models before and after applying the replacing rule, which meets the definition of model equivalence.

The equivalence between the model specifying $X \rightarrow Y$ and the model specifying $\mathrm{COV}(Ux,Uy)$ justifies the reverse application of the replacing rule, as long as the predictors of the effect variable are equal to or include those of the source variable after the change.

In applying the replacing rule, the recursiveness or nonrecursiveness of the model does not matter. As shown in Equations 1 and 2, the relational forms which the variables $P1$ through Pm or $Q1$ through Qn can take in the PBL are

[6] In CSM, variables are assumed to be measured from their means. Then, taking the expectation operator on the product of two variables (or vectors of variables) results in a covariance (or covariance matrix). Because the product of Equations 1 and 2 is algebraically the same as that of Equations 1 and 3, $\mathrm{COV}(X,Y)$ from Equations 1 and 2 is the same as that from Equations 1 and 3, given that $\mathrm{COV}(u,e)$ is non-zero.

not limited; they can be recursive or nonrecursive. Whether the PBL is recursive or nonrecursive does not affect the algebraic justification of the replacing rule. Whether the relationships in the succeeding block are recursive or nonrecursive is never a concern in applying the replacing rule. Thus, the recursiveness or nonrecursiveness of the model is of no concern in applying the replacing rule as long as the restriction of limited block-recursiveness holds.

A demonstration of applying the replacing rule will now be given. An initial, over-identified nonrecursive model is shown as Model 1A in Figure 1.

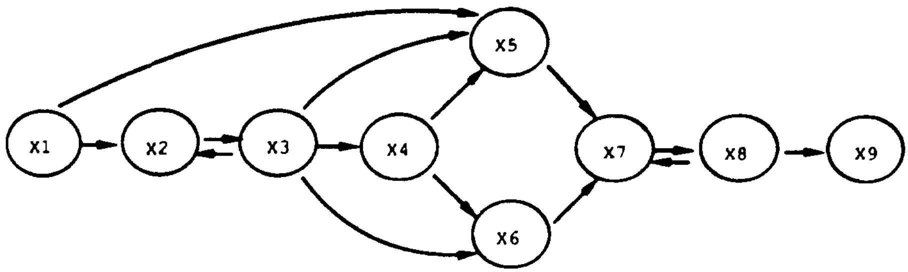
Let's focus on the four variables $X4$, $X5$, $X6$, and $X7$ in Model 1A to show the application of the replacing rule. It will be examined whether the replacing rule can be applied to the following focal blocks: ($X4$,$X5$), ($X5$,$X7$), and ($X4$,$X6$). A symbol U with appropriate subscript will stand for the residual of each variable.

Demonstration 1: Replacement of $X4 \rightarrow X5$ by $COV(U_4, U_5)$

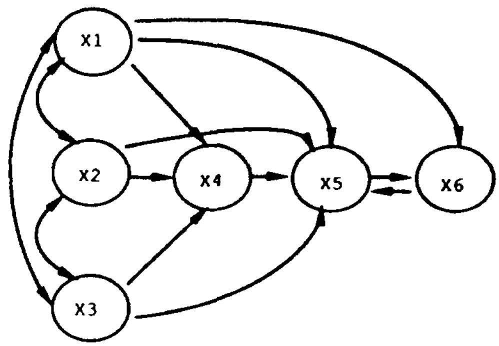
With the focal block of ($X4$,$X5$), limited block-recursiveness holds in Model 1A. The three blocks are as follows: ($X1$,$X2$,$X3$), ($X4$,$X5$), and ($X6$,$X7$,$X8$,$X9$). Recursiveness holds across the three blocks and within the focal block. Examining the predictors of the focal block, it is obvious that the effect variable $X5$ has the same predictor ($X3$) as the source variable $X4$, and one more predictor ($X1$). Thus $X4 \rightarrow X5$ can be replaced by COV(U_4,U_5), where the two-way arrow indicates a residual correlation. This type of modification is a generic case of applying the replacing rule and may be useful when the directional interpretation of the relationship between $X4$ and $X5$ is not substantively interesting. When the reverse application of the replacing rule is attempted, $X4$ should be the source and $X5$ the effect variable.

Demonstration 2: $X5 \rightarrow X7$ cannot be replaced by $COV(U_5, U_7)$

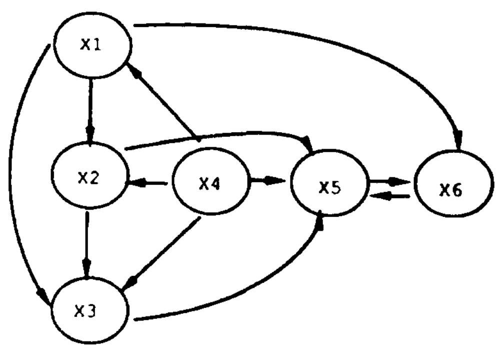
With the focal block of ($X5$,$X7$), model 1A is not block-recursive. Variable $X7$ has a nonrecursive relationship with $X8$. In defining a block, variables in a nonrecursive relationship should be grouped together so that recursiveness holds across blocks. Thus $X7$ and $X8$ should be in the same succeeding block. $X7$ and $X8$ do not comprise a focal block because they do not stand in a recursive relationship. Thus, ($X5$,$X7$) is not a legitimate focal block to apply the replacing rule. Even when $X7$ does not have a nonrecursive relationship with other variables, $X5 \rightarrow X7$ cannot be replaced by COV(U_5,U_7) because the predictors of $X7$ do not include those of $X5$.



Model 1A



Model 1B



Model 1C

Figure 1
Models to illustrate the replacing rule.

Demonstration 3: Replacement of X4→X6 by COV(U_4,U_6) or X4←X6

With $X4 \rightarrow X6$ as a focal block, limited block-recursiveness holds in Model 1A. Variable $X6$ has the same predictor ($X3$) as variable $X4$. Thus, the direct path $X4 \rightarrow X6$ can be replaced by COV(U_4,U_6). It is interesting to see a reverse application of the replacing rule in this case.

The covariance between U_4 and U_6 can be replaced by $X4 \rightarrow X6$ or $X4 \leftarrow X6$ because for either causal direction, the source variable and the effect variable have the same predictor ($X3$) in the PBL. Thus, this result represents a special case in applying the replacing rule: In a block-recursive system when variables in a focal block have the same predictor(s) in the PBL, the focal block is defined as a *symmetric focal block* and the variables as *symmetric* variables. In a symmetric focal block, inverting the path direction, replacing the direct path with a residual correlation, or replacing a residual correlation with a direct path of arbitrary direction between the symmetric variables generates a model equivalent to the initial model. This special case of applying the replacing rule to a symmetric focal block is very important in explaining Stelzl's (1986) rule 2 in a later section.

Application to a just-identified block

To illustrate another special case of applying the replacing rule, we will examine an application to a saturated or just-identified PBL. Saturation or just-identification in a PBL occurs when all potentially meaningful parameters are specified and thus there is no parsimony or restriction in the block. The parameters in a model (note that we mean structural model in this study) are path coefficients and variances/covariances of residuals. Because the identification and estimation of a PBL can be dealt with independently of the succeeding blocks in a block-recursive system, a PBL can be viewed as a model itself. Then a just-identified PBL can be considered as a just-identified model. A good discussion of just-identified recursive/nonrecursive models is given in Duncan (1975, Ch. 3 and Ch. 5).

Duncan (1969) presents nine equivalent models, all of which are just-identified recursive or nonrecursive models. When a just-identified model is restructured into another just-identified model, covariances among the variables do not change. This fact demonstrates that a just-identified PBL can be changed into another just-identified PBL without changing the covariances among the variables in the PBL. That is, the modified model with the new just-identified PBL is equivalent to the initial model.

Typically, a PBL is just-identified when any two variables are connected by either a direct path or residual correlation, but not by both; our replacing rule is limited to this type of just-identified block (JID block). In this case, all the variables in a JID block are completely connected. The path direction is arbitrary, and a direct path can be replaced by residual correlation[7], or a residual correlation can be replaced by a direct path in a JID block to generate an equivalent model. This procedure resembles the application of the replacing rule to symmetric focal block.

When equivalent models are generated by applying the replacing rule to a JID block, the JID block becomes a focal block. Because all variables can be completely connected by only correlations and all correlated variables are symmetric in a JID block, we can assume that all the variables in a JID block are symmetric variables. Then, changing the path direction and replacing a (residual) correlation with a direct path of arbitrary direction or replacing a direct path with (residual) correlation in a JID block is another way of applying the replacing rule on symmetric variables. This special case of applying the replacing rule on a JID block is very useful in explaining Stelzl's (1986) rule 1 in a later section.

An example of applying the replacing rule to a JID block is shown in models 1B and 1C of Figure 1. In Model 1B, variables $X1$, $X2$, $X3$, and $X4$ are defined as a PBL. Because any two variables are related either by a direct path or correlation, this PBL is just-identified. Recursiveness holds in this JID block, which becomes a focal block. The equations of variables $X5$ and $X6$ form the succeeding block dependent on this JID block. Thus, limited block-recursiveness holds in the model. The replacing rule can be applied on the JID block to generate equivalent models. Any correlation between two independent variables can be changed into a direct path of arbitrary direction and any direct path can be inverted or changed into a residual correlation within this block. One of the modified models according to the rule is shown in Model 1C, a path diagram which looks dramatically different from Model 1B. However, Model 1B and 1C are equivalent, that is, when they are fitted to a covariance matrix, they will generate identical fitted covariance matrices.

Before we close this section, we want to emphasize the usefulness of repeated applications of the replacing rule in generating equivalent models across a series of modified models. Once an equivalent model is generated by the application of the replacing rule, the rule can be applied again to the equivalent model to generate other equivalent models. For instance, A and B in A→B→C form a JID block and A←B→C is an equivalent model created by applying the replacing rule. The replacing rule can be applied to the focal block of B and C in A←B→C; since B and C form a JID block, and thus, A←B←C is another

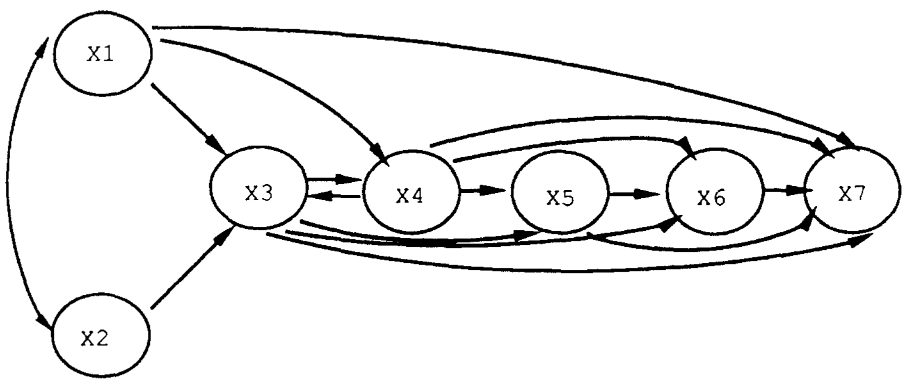
[7] A correlation between exogenous variables is a specific type of residual correlation if the exogenous variables are viewed as residuals. Thus, by (residual) correlation we refer to correlation of exogenous variables or correlation between errors in equations depending on the context.

equivalent model. The replacing rule was not applicable to the focal block of B and C in the initial model A→B→C. However, the block which includes B and C in the model A←B→C is a JID block and becomes the focal block to apply the replacing rule. As a result of repeated applications of the replacing rule, additional equivalent models can be generated.

Examples

To demonstrate the application of the replacing rule, a model is selected from the literature and modified so that applications of the replacing rule can be easily shown. The path analysis model of Locke, Frederick, Bobko, and Lee (1984) was selected. Their study investigated the relationship among self-efficacy, goals, task strategies and task performance. A variation of Locke et al.'s model is shown in Figure 2. Our purpose is only to demonstrate how to apply the replacing rule. We will show the application of the replacing rule on four relationships: *X*1 and *X*2; *X*5 and *X*6; *X*5 and *X*7; *X*6 and *X*7.

In Figure 2, *X*1 and *X*2 are correlated independent variables and form a JID block. Thus, the correlation between *X*1 and *X*2 can be replaced by a direct path of arbitrary direction as shown in Figure 3. Variables *X*5 and *X*6 are symmetric and form a symmetric focal block. The direct path *X*5→*X*6 can be replaced by *X*5←*X*6, or the residual correlation between them as shown in Figure 3.



X1: Ability
X2: Strategy Planning
X3: Strategies Used
X4: Post Training Performance
X5: Composite Self Efficacy
X6: Goal
X7: Performance

Figure 2
A variation of Locke, Frederick, Lee and Bobko's (1984) model.

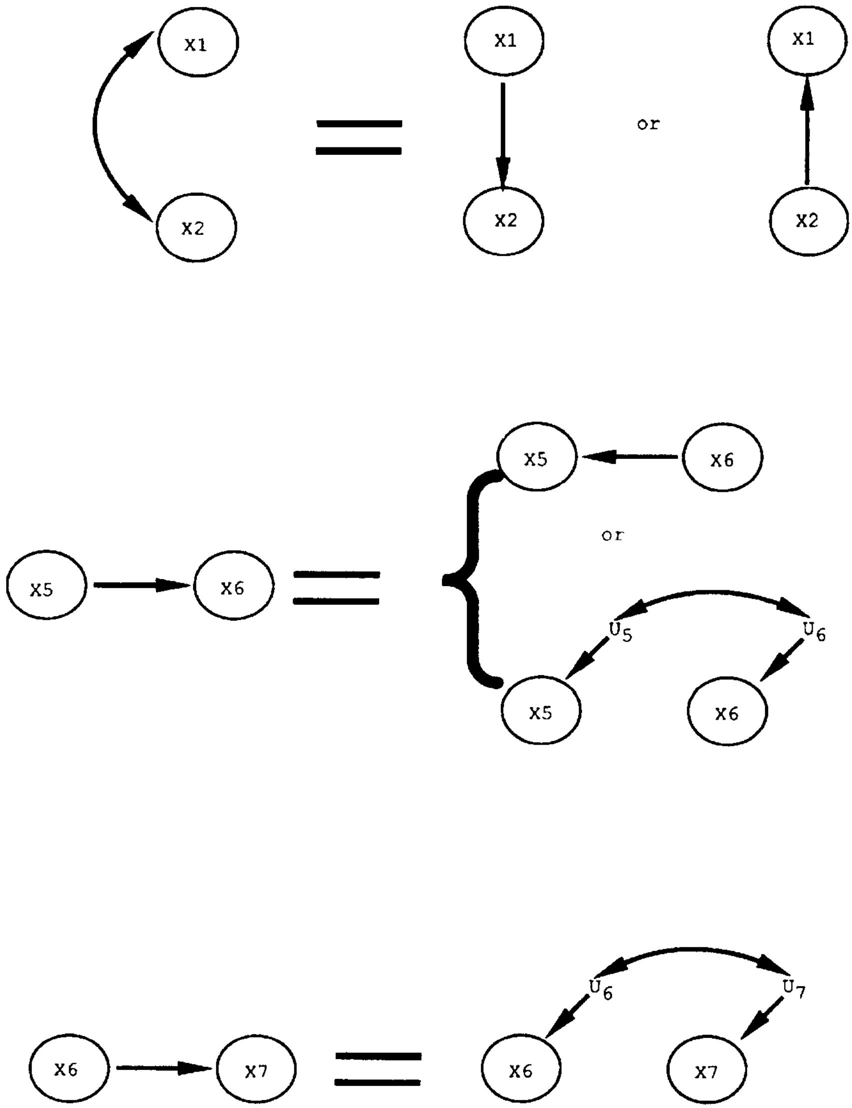


Figure 3
Applications of the replacing rule on the model

If *X*3 had a direct path instead of a reciprocal relationship with *X*4, variable *X*3 could form a JID block with *X*1 and *X*2. Then, the path directions among *X*1, *X*2, and *X*3 could be of arbitrary direction or any one of the direct paths could be replaced by a residual correlation.

With the focal block of $X6$ and $X7$, the requirement of limited block-recursiveness holds in the model of Figure 2. In the relationship between $X6$ and $X7$, the effect variable $X7$ has one more predictor ($X1$) in the PBL than its source variable, $X6$. Thus, the two variables in $X6 \rightarrow X7$ are not symmetric. However, this is a generic case of applying the replacing rule, and the direct path can be replaced by residual correlation as shown in Figure 3. With the block of $X5$ and $X7$ in Figure 2, block-recursiveness does not hold in the model. The relation between $X6$ and the block of $X5$ and $X7$ is not recursive; $X6$ is a dependent variable of $X5$ and is a predictor of $X7$ as well. Thus, the block of $X5$ and $X7$ is not a focal block to which application of replacing rule can be considered.

In the next section, a summary of Stelzl's (1986) four rules for generating equivalent models will be given, and how the replacing rule can provide a more general method for identifying equivalent models is presented.

Stelzl's Rules

Stelzl (1986) employs four recursive models to present each of her four rules: We will call her models Po, Qo, Ro, and So.

Stelzl's Rule 1

Stelzl's Rule 1 for recursive models will be introduced by model Po.

Model Po

Variables: $A \quad B \ldots E \ldots I.\ K \ldots P \ldots V$
Position: a b e i k p v
Zero paths: EK, IP
All other paths are non-zero-paths running from left to right.

To produce models equivalent to model Po, Stelzl's rule 1 posits the following: (a) interchange the variables on position 1 to k-1 or (b) interchange the variables on position k+1 to p-1 or (c) interchange the variables on position p+1 to v or (d) interchange the variables on position e to k. A zero-path indicates no path between two variables.

The variables in Stelzl's (1986) rule (a), (b), and (c) can be categorized into three sets of variables: The first set of variables is located before the end-point of the first zero-path; the second set of variables between two neighboring end-points of zero-paths; and the third set of variables after the end-point of the last zero-path.

Stelzl's (1986) rule 1 will now be explained in terms of the replacing rule. Each set of variables in model Po can be viewed as a block in a block-recursive system. The set of the variables before the end-point of the first zero-path forms a JID block, because all variables in this block are completely connected. Thus, the direction of any path in this block can be inverted to generate an equivalent model according to the replacing rule. Variables in the second set or in the third set form symmetric focal blocks by our definition because they have direct paths with all the variables in the previous block(s). Then, the direct path between any two variables within these two sets can be inverted arbitrarily according to the replacing rule. Inverting the path direction in the three blocks does not change the recursiveness of the relationships in each block, although the position of the variables must be changed in order for the path diagram to appear recursive. An example will now be shown.

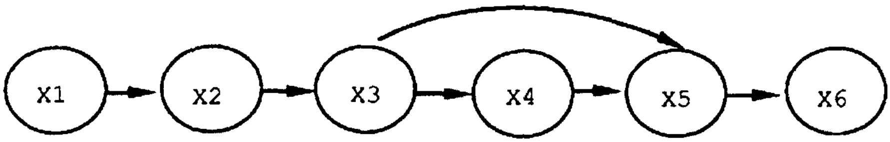
Suppose a simple path diagram as in Model 4A of Figure 4.

In Model 4A, $X1$ and $X2$ form a JID block and $X4$ and $X5$ are symmetric. First, the path directions between $X1$ and $X2$ can be inverted. Next, the path direction between $X4$ and $X5$ in the symmetric focal block can be inverted. These two modifications are shown in Model 4B of Figure 4 (next page). Inverting the path directions in this way does not alter the recursiveness of the model. The positions of the variables can be adjusted so that the model appears recursive as in Model 4C. Changing the position of variables is another way of inverting path directions. Thus we do not differentiate between inverting path directions and changing positions of variables although Stelzl (1986) prefers the phrase *changing positions of variables* in her rules 1 and 2. Thus, the replacing rule, applied to recursive models, provides the same results as Stelzl's rule (a), (b), and (c) in generating equivalent models.

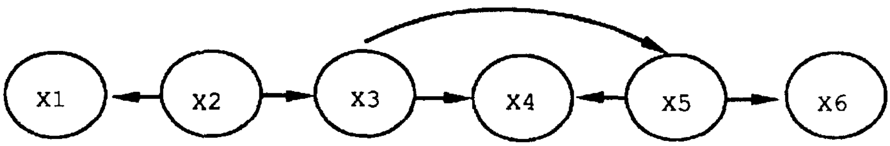
Stelzl's (1986) rule (d) will now be discussed. The zero-path between E and K is the first zero-path in model Po. In the JID block of model Po, variable E can be moved to any position that keeps its zero-path with variable K. Suppose variable E is moved to position k-1, adjacent to variable K. Then, the variables on position 1 to k-2 and variable K form a new JID block. This is an example of a repeated application of the replacing rule. The variables on position 1 to k-2 and variable E comprise the original JID block. In a JID block, the position of the variables is arbitrary. Because the new JID block and original JID block share the first k-2 positions, variable E (or K) can be moved to any of the first k-2 positions and to position k (or k-1). Thus, the position of variable K can be on position e and that of variable E can be on position k, which also results by applying Stelzl's rule (d).

Stelzl's Rule 2

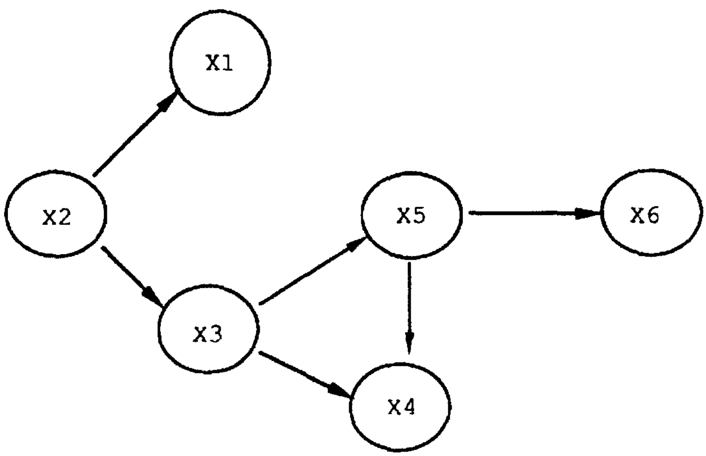
Stelzl's rule 2 for recursive models will be introduced by model Qo.



Model 4A



Model 4B



Model 4C

Figure 4
Maintenance of model recursiveness after inversion of path direction.

Model Qo

Variables: $A\ B \ldots D\ E\ F \ldots L\ M\ N \ldots Q$
Zero-paths: EM, EN
All other paths are non-zero paths.

To produce models equivalent to model Qo, Stelzl's rule 2 posits a change in the positions of *M* and *N*: The adjacent variables *M* and *N* are interchangeable as long as they have zero-paths originating from the same preceding variable(s), regardless of the number of preceding variables simultaneously having zero-paths to *M* and *N*.

Stelzl's rule 2 will now be explained in terms of the replacing rule. Variables *M* and *N* are symmetric in a focal block if model Qo is viewed as a block-recursive system. Thus, the replacing rule allows changing $M \rightarrow N$ into $M \leftarrow N$. $M \leftarrow N$ is equivalent to the position change of *M* and *N* as posited by Stelzl's rule 2.

Stelzl's Rule 3 and 4

Stelzl's rule 3 for recursive models will be introduced by model Ro.

Model Ro

Variables: $A\ B \ldots E \ldots M\ N \ldots Q$
N has non-zero-paths from all preceding variables.
All other paths may be zero-paths or non-zero-paths.

To produce models equivalent to Ro, Stelzl's rule 3 posits that a non-zero-path between *N* and any preceding variable, for example *M*, can be replaced by a residual correlation between the two variables. The reversal of this procedure also yields an equivalent model. When *N* is related to all of its predetermined variables by either a direct path or a residual correlation, the residual correlation between *N* and any preceding variable (e.g., *M*) can be replaced by a direct path, $M \rightarrow N$. A comparison of this rule with the replacing rule will be given after Stelzl's rule 4 is introduced.

Stelzl's rule 4 for recursive models will be introduced by model So.

Model So

Variables: $A\ B \ldots D\ E\ F \ldots L\ M\ N \ldots Q$
Zero-paths: EM, EN
M and *N* should be adjacent and are connected by a direct path.
M may have zero-path(s) or non-zero-path(s) from the preceding variables.
N has non-zero paths from all the preceding variables but *E*.
All other paths may be zero paths or non-zero paths.

To produce models equivalent to model So, Stelzl's rule 4 posits that the direct path from M to N can be replaced by the residual correlation between the two variables; when N has more than one zero-path with its preceding variables, M must have the same zero-paths as, or more zero-paths than, N has with the preceding variables. Conversely, the correlated residual between M and N can be replaced by a direct path from M to N if M and N are adjacent (M before N) and N has zero-paths with only the predetermined variables with which M also has zero-paths.

Stelzl's rule 3 and 4 can be described in terms of the replacing rule. The relationship between M and N can be viewed as a focal block in block recursive systems Ro and So. In this focal block, variable M is the source variable and N the effect variable. All the variables before this focal block form the PBL. In model Ro, the effect variable N has more predictors in the PBL than the source variable M. In model So, the effect variable N has the same predictors in the PBL as the source variable M. The replacing rule deals with the two situations offered by models Ro and So; if the effect variable has the same predictors as or more predictors than the source variable, then the direct path from the source variable to the effect variable can be replaced by a residual correlation. Likewise, the reverse application of the replacing rule explains the reverse application of Stelzl's rules 3 and 4. Thus, the replacing rule subsumes Stelzl's rules 3 and 4 when it is applied to recursive models.

Conclusion

The comprehensiveness of the replacing rule has been discussed and compared with Stelzl's four rules. The replacing rule, when applied to a recursive model, combines Stelzl's rules 3 and 4. Application of the replacing rule to a JID block in a recursive model provides the same results as part of Stelzl's rule 1, and application of the replacing rule to a symmetric focal block in a recursive model provides the same results as Stelzl's rule 2 and part of her rule 1.

In addition to its theoretical comprehensiveness, the replacing rule has a broader applicability with simpler conditions for application. Once limited block-recursiveness holds in a model, there is a very simple condition that must be met in order to use the replacing rule on a focal block: An effect variable has the same predictors as, or more predictors than, the source variable has in the PBL. Since the publication of Stelzl's (1986) rules, little application has been made of them in empirical research. A possible reason for this neglect may lie in the complexity of description required in explaining models Po, Qo, Ro, and So, which must be examined by the researcher before application of the rules is considered.

Two special cases of applying the replacing rule were discussed: Symmetric variables in a focal block and a JID block. They are very useful in generating equivalent models that can provide logically/substantively different interpretations.

Ease of drawing equivalent models

The ease of drawing equivalent models with the replacing rule encourages its use in practice. We introduced the concept of inverting path direction by the replacing rule. In our discussion, we emphasized the preferability of the replacing rule compared to the position change of variables in Stelzl's rules 1 and 2. The method of inverting path directions prevents mistakes which an unwary investigator might commit in generating equivalent models.

In making position changes according to Stelzl's rules, it is critical that the zero-paths and non-zero-paths that a particular variable has with others should not change after the position of the variable has changed. For instance, in a simple model A→B→C, Stelzl's rule 1 allows a change in the positions of A and B to generate equivalent models. When the positions of A and B are changed, the new model should appear as B→A C, not B→A→C. Because A has a zero-path to C and B has a non-zero path to C in the initial model, these relationships should not be changed in the new model. However, it is very easy for a novice to be tempted to draw B→A→C, which is not equivalent to A→B→C. When the replacing rule is applied, this type of mistake is easily prevented. According to the replacing rule, A and B form a JID block in A→B→C; thus, the path direction can be inverted. The new model appears as A←B→C, another form of B→A C, which can also be derived by a *correct* application of Stelzl's rule 1. Inverting the path direction prevents possible mistakes in changing the positions of variables.

Broad opportunity with a JID block

We would like to emphasize the role of a JID block in generating equivalent models. Because just-identification can occur in a nonrecursive PBL as well as in a recursive PBL, if a systematic method is known to change a recursive JID block to a nonrecursive JID or a nonrecursive JID to a recursive JID block, or from a nonrecursive JID block to another nonrecursive one, that method will allow us to deal with a focal block that is JID and nonrecursive. The replacing rule allows one to use a JID block to generate equivalent models.

Resolving the indeterminacy of theories

Finally, it must be considered, in light of the existence of multiple equivalent models, how one resolves the indeterminacy underlying the distinct models that generated the same data. Data alone cannot provide such a resolution, for it is obvious that data or the statistics of fit measures do not distinguish equivalent models and thus do not inform or dictate any theory. Data normally supports multiple alternatives. Thus, one should be equipped with some rationale to rule out some causal connections as implausible. During specification searches, very often the MI of a path coefficient between two variables is identical to that of residual correlation between them; freeing either parameter may result in two equivalent models. If one is confident that all causal variables are specified in the model, one would choose freeing the path instead of freeing the residual correlation. If the relationship between the two variables is not theoretically interesting or seems to be spurious, the residual correlation should be chosen.

If one generates equivalent models by the application of a priori rules, the following two conditions can be used to determine a better model among equivalent models: Time precedence and mediating mechanisms. Time precedence is cited as one of the conditions that exists between a cause and an effect (Cook & Campbell, 1979; James et al., 1982; Kenny, 1979). Effects follow causes in time. Causes do not run backwards in time, so some causal orderings are ruled out for variables measured at different time points.

Mediating mechanisms are involved in most strong causal propositions, that is, the molar laws of Cook and Campbell (1979). According to Cook and Campbell, the term "molar laws" refers to "causal laws stated in terms of large and often complex objects" (p. 32); for example, weather condition has an effect on work performance. Mediating mechanism refers to the specified causal connections "at a level of smaller particles than make up the molar objects and on a finer time scale" (Cook & Campbell, 1979, p. 32); for example, the mediating mechanism between weather condition and work performance is the psychological state of perceiving the weather and adjusting one's motivation to work depending on the quality of the weather. If we specify the mediating mechanism as weather condition → psychological state → work performance, the path coefficient from the weather condition to work performance should be fixed at zero. Similarly, any causal connection should be "dependent on a theoretical rationale involving several mediating mechanisms" (James et al., 1982, p. 35). Mediating mechanisms help to limit the acceptable causal connections among complex variables in a molar law.

By the examination of time precedence and mediating mechanisms, it is hoped that at least the number of equivalent models is less than infinity. However, we still may not know at the moment, when faced with multiple equivalent models, how the structural process in each model captures the partial image of the whole picture. Therefore, it is advisable to retain multiple models if they are not falsified by data or by present theoretical rationale, until each of them is more specifically examined by more refined theories. Mulaik (1987) argues that "progress in science benefits not from banning theorizing and hypothesizing but from the existence of numerous rival hypotheses and theories, even if they be bizarre, false, and currently untestable . . . " (p. 21). When the optimal model is not easily determined, suspending the decision and retaining the multiple models is suggested until more rigorous evidence is accumulated through experiments, longitudinal studies, or other investigative means.

One last suggestion in evaluating multiple alternative theories is to consider parsimony (Mulaik et al., in press). Some models have more conditions by which they could be falsified than others. The falsifiability of a model increases as fewer parameters are specified to be estimated from the data. Thus, among equivalent models, the one that specifies fewest parameters, that is, the most parsimonious model, has been subject to the most stringent test. This most parsimonious model should be chosen over less parsimonious models if they are equivalent models. However, this criterion of parsimony is not helpful for evaluating the equivalent models, generated by a priori rules or through identical MIs because these procedures do not change the number of parameters. However, we do not rule out the possibility that equivalent models can have different numbers of parameters. If multiple equivalent models do have differing numbers of parameters, then parsimony would be the most useful criterion.

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