Confirmatory Factor Analysis

James H. Steiger

Department of Psychology and Human Development
Vanderbilt University

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Introduction

- In classical factor analysis, an initial "unrotated" factor pattern is rotated to "simple structure" either by hand or by an algorithm.
- The algorithms have been improving radically in recent years, due to work at UCLA by Jennrich, Bentler, and their associates.
- These new technical developments have produced further improvements in the quality of rotation methods.
- The interested reader should consult the review article by Browne (2001), the recently published chapters by Mulaik (2010, chapters 11 and 12, and references therein), and the work on bifactor rotation by Jennrich and Bentler.

Introduction

- A major goal of factor analysis is to produce economy of description — a large number of observed variables are explained by a much smaller number of common factors, as evidenced by lots of near-zero values in the factor pattern.
- In keeping with the desire to keep the "structure simple," factor analysis programs often "clean up" the rotated pattern by blanking out values that are smaller than some specified value.
- This practice makes some users uneasy:
  1. Are the blanked-out loadings statistically significant?
  2. If not, should they be treated as zero?
  3. What about the accept-support fallacy?

Introduction

- These problems would be difficult enough if standard errors for the factor loadings were available.
- However, the rotated loadings have standard errors that are not given by most programs. Jennrich developed theory for calculating these loadings a long time ago, but the computational difficulties have deterred many authors from including them in exploratory factor analysis packages.
- Without an index of variability, one cannot determine whether loadings are significant.
- At least two modern programs, *Mplus* and *CEFA* provide standard errors for rotated loadings. Mplus is commercial software, while CEFA is freeware.
- In this module, we examine an alternative, widely-used approach to cleaning up a factor pattern and getting statistical indices concerning the effectiveness of our efforts.
- This approach is called *confirmatory factor analysis*.
- Although they all use the same basic technology, there are several distinctly different approaches to confirmatory factor analysis that we'll summarize in the next sections.

Classic Approaches to Confirmatory Factor Analysis

- There are several primary approaches to confirmatory factor analysis:
  1. Pure Confirmatory Factor Analysis (perhaps followed by model modification, often using "modification indices.")
  2. Confirmatory Factor Analysis from an Exploratory Pattern based on a fixed cutoff for "trivial loadings" to be eliminated.
  3. Statistically-Guided "Exploratory-Confirmatory" Factor Analysis based on a "reference pattern." (Jöreskog, 1978)

Classic Approaches to Confirmatory Factor Analysis

Pure Confirmatory Factor Analysis

- In this most fundamental approach, the researcher approaches the analysis with a very firm idea of how the factors are supposed to align with the variables.
- The researcher's hypothesis is expressed as a hypothesized pattern form, containing fixed values of zero and free parameters for loadings that are hypothesized to be consequential.

Classic Approaches to Confirmatory Factor Analysis

Pure Confirmatory Factor Analysis

Example (Confirmatory Factor Analysis)

In our hypothetical Athletics data set, we might imagine that there were no cross-loadings, and start with the hypothesized pattern as follows:

$$\mathbf{\Lambda} = \begin{bmatrix} \theta_1 & 0 & 0 \\ \theta_2 & 0 & 0 \\ \theta_3 & 0 & 0 \\ 0 & \theta_4 & 0 \\ 0 & \theta_5 & 0 \\ 0 & \theta_6 & 0 \\ 0 & 0 & \theta_7 \\ 0 & 0 & \theta_8 \\ 0 & 0 & \theta_9 \end{bmatrix}$$

Classic Approaches to Confirmatory Factor Analysis

Pure Confirmatory Factor Analysis

Example (Confirmatory Factor Analysis)

Assuming orthogonal factors, the hypothesis would be completed by including the diagonal matrix $\mathbf{U}^2$ containing elements $\theta_1 0$ through $\theta_1 8$ on its diagonal.

As we shall see later, this leads to a hypothesized structure for the covariance matrix that is a function of the parameters and the way they are placed in the model matrices. Maximum likelihood estimates for the free parameters can be obtained, and a χ^2 test of perfect model fit performed.

Precisely how this χ^2 test should be processed is still the subject of some discussion and controversy, and will be dealt with in detail later.

Classic Approaches to Confirmatory Factor Analysis

Pure Confirmatory Factor Analysis

- Should the model prove inadequate in obvious ways that seem exceed chance variation, the researcher will often update the model. We'll digress now to examine how this might be done, with an example.

Classic Approaches to Confirmatory Factor Analysis

Confirmatory FA from an Exploratory Pattern

- In this approach, one cleans up a final rotated pattern, setting to zero all loadings that are below a certain cutoff.
- One then fits the resulting pattern with CFA.
- We'll work an example with the Athletics data set.

Classic Approaches to Confirmatory Factor Analysis

Jöreskog's Exploratory-Confirmatory Approach

- A fundamental problem with using most exploratory factor analysis routines with a CFA framework is the absence of standard errors.
- J oreskog (1978) offered an ingenious partial workaround for this problem.
  1. Perform an exploratory factor analysis, and decide on the number of factors, m. In many textbook examples, the decision is relatively clear cut. Be forewarned - in practice the decision may be quite difficult.
  2. Fit an m-factor model, and rotate to simple structure using varimax or promax. (In the original article, Jöreskog said to use promax.)
  3. For each column of the factor pattern, find the largest loading, then constrain all the other loadings in that row to be zero, and fit the resulting model as a confirmatory factor model. This confirmatory model will have exactly the same discrepancy function and χ^2 value as the exploratory factor analysis that preceded it.
  4. Examine the factor pattern, and test all factor loadings. Delete "non-significant" loadings from the model. After checking the fit, the user can decide whether to terminate the process, or look for more loadings to delete.
- I've automated this process. Let's try a demo.