

# Homework 1

Psychology 312

- (12 points) Suppose you have a set of data in the variable  $X$  having a sample mean  $\bar{x}_\bullet = 100$  and a sample standard deviation  $S_X = 10$ . For each of the following transformed variables, indicate the mean  $\bar{y}_\bullet$  and the standard deviation  $S_Y$ .
  - $y_i = 2x_i$
  - $y_i = x_i - 5$
  - $y_i = (x_i - \bar{x}_\bullet)/S_X$
  - $y_i = 2(x_i - 5)/10 + 7$
- (6 points). A set of scores that are in  $Z$ -score form has a mean of 0 and a standard deviation of 1. Suppose they are multiplied by 10, then 5 is added to each number. What will be the mean and standard deviation of the resulting scores?
- (16 points). You have two sets of scores  $X$  and  $Y$ , on the same  $N$  individuals. Suppose  $\bar{x}_\bullet = 34.5$ ,  $\bar{y}_\bullet = 44.9$ ,  $S_X^2 = 38.8$ ,  $S_Y^2 = 44.4$ , and  $S_{XY} = 20$ .
  - Compute the mean and variance of the linear combination scores  $w_i = 2x_i - y_i$ .
  - Compute the covariance and correlation between the two linear combinations  $a_i = x_i + y_i$  and  $b_i = x_i - 2y_i$ .
- (8 points). The grades in a particular course have a mean of 70 and a standard deviation of 10. However, they are supposed to have a mean of 65 and a standard deviation of 8. You and a friend are the teaching assistants in the course, and are asked to transform the grades. You decide to multiply each grade by .8, then add 9 to each grade. You are about to do this when your friend interrupts you, and says that you should *first* add 11.25 to each score, and *then* multiply by .8. Who is correct?
- (8 points). Given random variables  $X$  and  $Y$ , suppose it is known that both random variables have zero means, and that  $\mathcal{E}(X^2) = 9$ ,  $\mathcal{E}(Y^2) = 4$ , and that  $\mathcal{E}(XY) = 4$ . Find the covariance and correlation between  $X$  and  $Y$ , i.e.,  $\rho_{xy}$  and  $\sigma_{xy}$ .

6. (20 points). Given the following matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 9 \\ 0 & 6 & 7 \\ 3 & 3 & 8 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 3 & 13 \\ 2 & 2 & 4 \\ 3 & 1 & 7 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 6 & 7 & 5 \\ 6 & 8 & 6 \\ 15 & 19 & 11 \end{bmatrix}$$

Compute the following:

- (a)  $\mathbf{A} + \mathbf{B}$
- (b)  $\mathbf{C}\mathbf{C}'$
- (c)  $\mathbf{A} - \mathbf{C}$
- (d)  $\text{Tr}(\mathbf{A}\mathbf{A}')$
- (e)  $\text{Tr}(\mathbf{A}'\mathbf{C})$

7. (5 points). Which descriptor below is *not* true of this matrix?  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

- (a) Square matrix
- (b) Scalar matrix
- (c) Diagonal matrix
- (d) Symmetric matrix
- (e) Lower triangular matrix