# Statistical Models in Structural Equation Modeling

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## *Models and Methods*

Structural equation models have achieved increasing popularity in the social sciences. Much of the credit for this popularity can be attributed to the flexibility and power of the methods themselves. Equally important has been the availability of computer software for performing the modeling process.

An enormous amount of material has been written on structural modeling. There are now numerous textbooks and monographs for the beginner. All of these books have significant virtues. The reader with a serious interest in the subject should probably at least browse through several of these books.

For a very interesting debate on the value of structural models in the social sciences, the *Summer 1987* issue of the *Journal of Educational Statistics* is strongly recommended. This issue contains a critique of path analysis by D.A. Freedman, and responses to that critique by a number of writers.

Discussion of the deeper apects of the theoretical connections between causal inference and statistical modeling is beyond the scope of this chapter. I also assume that the reader has basic familiarity with the terminology of path diagrams.

### The LISREL Model

This section begins with a review of several important models for the analysis of covariance structures. In the following discussion, all variables will be assumed to be in deviation score form (i.e., have zero means) unless explicitly stated otherwise.

In his 1986 review chapter on developments in structural modeling, Bentler described 3 general approaches to covariance structure representations. The first and most familiar involved integration of the psychometric factor analytic (FA) tradition with the econometric simultaneous equations model (SEM). This approach, originated by a number of authors including Keesling, Wiley, and Jöreskog was described by Bentler with the neutral acronym FASEM. The well-known LISREL model is of course the best known example of this approach.

The LISREL model can be written in three interlocking equations. Perhaps the key equation is the structural equation model, which relates latent variables.

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta} \tag{1}$$

The endogenous, or "dependent" latent variables are collected in the vector $\boldsymbol{\eta}$, while the exogenous, or "independent" latent variables are in $\boldsymbol{\xi}$. $\mathbf{B}$ and $\boldsymbol{\Gamma}$ are coefficient matrices, while $\boldsymbol{\zeta}$ is a random vector of residuals, sometimes called "errors in equations" or "disturbance terms." The elements of $\mathbf{B}$ and $\boldsymbol{\Gamma}$ represent path coefficients for directed relationships among latent variables. It is assumed in general that $\boldsymbol{\zeta}$ and $\boldsymbol{\xi}$ are uncorrelated, and that $\mathbf{I} - \mathbf{B}$ is of full rank.

Because usually $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ are not observed without error, there are also factor model (or "measurement model") equations to account for measurement of these latent variables through manifest variables. The "measurement models" for the two sets of latent variables are

$$\mathbf{y} = \boldsymbol{\Lambda}_y \boldsymbol{\eta} + \boldsymbol{\varepsilon} \tag{2}$$

and

$$\mathbf{x} = \boldsymbol{\Lambda}_{\mathbf{x}} \boldsymbol{\xi} + \boldsymbol{\delta} \tag{3}$$

With

$$\boldsymbol{\Sigma} = \left[\begin{array}{c|c} \boldsymbol{\Sigma}_{yy} & \boldsymbol{\Sigma}_{yx} \\ \hline \boldsymbol{\Sigma}_{xy} & \boldsymbol{\Sigma}_{xx} \end{array}\right] \tag{4}$$

the LISREL model is that

$$\boldsymbol{\Sigma}_{yy} = \boldsymbol{\Lambda}_y \left(\mathbf{I} - \mathbf{B}\right)^{-1} \left(\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Gamma}' + \boldsymbol{\Psi}\right)\left(\mathbf{I} - \mathbf{B}'\right)^{-1} \boldsymbol{\Lambda}_y' + \boldsymbol{\Theta}_\epsilon \tag{5}$$

$$\boldsymbol{\Sigma}_{xx} = \boldsymbol{\Lambda}_x \boldsymbol{\Phi} \boldsymbol{\Lambda}_x' + \boldsymbol{\Theta}_\delta \tag{6}$$

$$\boldsymbol{\Sigma}_{xy} = \boldsymbol{\Lambda}_x \boldsymbol{\Phi}\boldsymbol{\Gamma}' \left(\mathbf{I} - \mathbf{B}'\right)^{-1} \boldsymbol{\Lambda}_y' \tag{7}$$

$\boldsymbol{\Phi}, \boldsymbol{\Psi}, \boldsymbol{\Theta}_\epsilon$, and $\boldsymbol{\Theta}_\delta$ are the covariance matrices for $\boldsymbol{\xi}, \boldsymbol{\zeta}, \boldsymbol{\varepsilon}$, and $\boldsymbol{\delta}$ respectively.

There seems to be considerable confusion in the literature about the precise assumptions required for Equations 1 through 7 to hold. Jöreskog and Sörbom (1989) state the assumptions that (1) $\boldsymbol{\zeta}$ is uncorrelated with $\boldsymbol{\xi}$, (2) $\boldsymbol{\varepsilon}$ is uncorrelated with $\boldsymbol{\eta}$, (3) $\boldsymbol{\delta}$ is uncorrelated with $\boldsymbol{\xi}$, (4) $\boldsymbol{\zeta}, \boldsymbol{\varepsilon},$ and $\boldsymbol{\delta}$ are mutually uncorrelated, and (5) $\mathbf{I} - \mathbf{B}$ is of full rank. However, it appears that Equation 7 also requires an assumption not stated by Jöreskog and Sörbom (1989), i.e., that $\boldsymbol{\varepsilon}$ and $\boldsymbol{\xi}$ are uncorrelated.

This model reduces to a number of well-known special cases. For example, if there are no $y$-variables, then the model reduces to the common factor model, as can be seen from Equation 6.

An important aspect of the LISREL approach is that, in using it, variables must be arranged according to type. Manifest and latent, "exogenous" and "endogenous" variables are used in different places in different equations. Moreover, LISREL's typology for manifest variables is somewhat different from that used by other models. Specifically, in LISREL a manifest variable is designated as $x$ or $y$ on the basis of the type (exogenous or endogenous) of latent variable it loads on.

It is, of course, possible to translate models from a path diagram representation of a model to a LISREL model. However, this is not always easy. In some well known cases special strategies must be used to "trick" the LISREL model into analyzing a path diagram representation. For example, the LISREL equations do not explicitly include direct representation of a path in which an arrow goes from a manifest exogenous variable to a latent endogenous variable. Consequently a dummy latent exogenous variable (identical to the manifest variable) must be created in such cases.

In his review, Bentler (1986) referred to the models of McArdle (1978) and Bentler and Weeks (1979) as "generic" approaches, in that their emphasis was on the distinction between independent (exogenous) and dependent (endogenous) variables, rather than manifest and latent variables.

McArdle (1978) proposed an approach that was considerably simpler than the LISREL model. This approach, in essence, did not require any partitioning of variables into types. One could represent all paths in only two matrices, one representing directed relationships among variables, the other undirected relationships. McArdle's approach, which he called the RAM model, could be tested easily as a special case of McDonald's COSAN model.

McArdle's specification was innovative, and offered substantial benefits. It allowed path models to be grasped and fully specified in their simplest form — as linear equations among manifest and latent variables. Instead of 18 model matrices, and a plethora of different variable types, one only needed 3 matrices! After reading some of McArdle's early papers, I was motivated to seek an automated approach to structural modeling. Ironically, it took some time for McArdle's work to receive the attention it deserved. The work initially met with a lukewarm reception from journal editors and rather harsh opposition from some reviewers. It took 4 years for a detailed algebraic treatment (McArdle & McDonald, 1984) to pass through the review process and achieve publication. By then, unfortunately, the full credit due to McArdle had been diluted.

## The COSAN Model

This section begins with a brief description of the McDonald's COSAN model. Let $\mathbf{\Sigma}$ be a population variance-covariance matrix for a set of manifest variables. The COSAN model (McDonald, 1978) holds if $\mathbf{\Sigma}$ may be expressed as

$$\mathbf{\Sigma} = \mathbf{F}_1\mathbf{F}_2 \dots \mathbf{F}_k\mathbf{P}\mathbf{F}_k{}' \dots \mathbf{F}_2{}'\mathbf{F}_1{}' \tag{8}$$

where $\mathbf{P}$ is symmetric and Gramian, and any of the elements of any $\mathbf{F}$ matrix or $\mathbf{P}$ may be constrained under the model to be a function of the others, or to be specified numerical values. As a powerful additional option, any square $\mathbf{F}$ matrix may be specified to be the *inverse* of a patterned matrix. This "*patterned inverse*" option is critical for applications to path analysis. A COSAN model with $k$ $\mathbf{F}$ matrices is referred to as "a COSAN model of order $k$."

Obvious special cases are: Orthogonal and oblique common factor models, confirmatory factor models, and patterned covariance matrices.

McDonald's COSAN model is a powerful and original approach which offers many benefits to the prospective tester of covariance structure models. Testing and estimation for the model were implemented in a computer program called, aptly enough, COSAN (See Fraser and McDonald, 1988 for details on a recent version of this program, which has been available since 1978).

In 1978, J. J. McArdle proposed some simple rules for translating any path diagram directly to a structural model. In collaboration with McDonald, he proposed an approach which yielded a model directly testable with the COSAN computer program.

## McArdle's RAM Model

McArdle's approach is based on the following covariance structure model, which he has termed the RAM model:

Let $\mathbf{v}$ be a $(p+n)\times 1$ random vector of $p$ manifest variables and $n$ latent variables in the path model, possibly partitioned into manifest and latent variables subsets in $\mathbf{m}$ and $\mathbf{l}$, respectively, in which case

$$\mathbf{v} = \begin{bmatrix} \mathbf{m} \\ \hline \mathbf{l} \end{bmatrix} \tag{9}$$

(This partitioning is somewhat convenient, but not necessary.) For simplicity assume all variables have zero means. Let $\mathbf{F}$ be a matrix of multiple regression weights for predicting each variable in $\mathbf{v}$ from the $p+n-1$ other variables in $\mathbf{v}$. $\mathbf{F}$ will have all diagonal elements equal to zero. In general, some elements of $\mathbf{F}$ may be constrained by hypothesis to be equal to each other, or to specified numerical values (often zero). Let $\mathbf{r}$ be a vector of latent exogenous variables, including residuals. The path model may then be written

$$\mathbf{v} = \mathbf{F}\mathbf{v} + \mathbf{r} \tag{10}$$

In path models, all endogenous variables are perfectly predicted through the arrows leading to them. Since endogenous variables are dependent variables in one or more linear equations, their variances and covariances can be determined from the variances and covariances of the variables with arrows pointing to them. Ultimately, the variances and covariances of *all endogenous variables* are explained by a knowledge of the linear equation set up and the variances and covariances of exogenous variables in the system. Consequently, elements of $\mathbf{r}$ corresponding to endogenous variables in $\mathbf{v}$ will be null. The matrix $\mathbf{F}$ contains the regression coefficients normally placed along the arrows in a path diagram. $f_{ij}$ is the path coefficient from $v_j$ to $v_i$. If a variable $v_i$ is exogenous, i.e., has no arrow pointing to it, then row $i$ of $\mathbf{F}$ will be null, and $r_i = v_i$. Hence, the non-null elements of the variance covariance matrix of $\mathbf{r}$ will be the coefficients in the "undirected" relationships in the path diagram.

Define $\mathbf{P} = E(\mathbf{rr}')$. Furthermore, let $\mathbf{W} = E(\mathbf{vv}')$, and $\boldsymbol{\Sigma} = E(\mathbf{mm}')$. The implications of Equation 10 for the structure of $\boldsymbol{\Sigma}$, the variance-covariance matrix of the manifest variables, can now be derived. Regardless of whether the manifest and latent variables were partitioned into distinct subsets in $\mathbf{v}$, it is easy to construct a "filter matrix" $\mathbf{J}$ which carries $\mathbf{v}$ into $\mathbf{m}$. If the variables in $\mathbf{v}$ are partitioned into manifest and latent variables, one obtains

$$\mathbf{J} = \left[\mathbf{I} \;\vdots\; \mathbf{0}\right] \tag{11}$$

$$\mathbf{m} = \mathbf{Jv} \tag{12}$$

and consequently

$$\boldsymbol{\Sigma} = E(\mathbf{mm}') = \mathbf{J}E(\mathbf{vv}')\mathbf{J}' = \mathbf{JWJ}' \tag{13}$$

Note that, since **I - F** is by assumption nonsingular, Equation 10 may be rewritten in the form

$$\mathbf{v} = (\mathbf{I} - \mathbf{F})^{-1}\mathbf{r} \tag{14}$$

one obtains

$$\mathbf{W} = (\mathbf{I} - \mathbf{F})^{-1}\mathbf{P}(\mathbf{I} - \mathbf{F})^{-1\prime} \tag{15}$$

Equations 13 and 15 imply

$$\boldsymbol{\Sigma} = \mathbf{J}(\mathbf{I} - \mathbf{F})^{-1}\mathbf{P}(\mathbf{I} - \mathbf{F})^{-1\prime}\mathbf{J}' \tag{16}$$

This shows that any path model may be written in the form

$$\boldsymbol{\Sigma} = \mathbf{F}_1\mathbf{F}_2\mathbf{P}\mathbf{F}_2{}'\mathbf{F}_1{}' \tag{17}$$

as a COSAN model of order 2, where (assuming the manifest and latent variables are stacked in separate partitions)

$$\mathbf{F}_1 = \mathbf{J} = \left[\mathbf{I} \;\vdots\; \mathbf{0}\right] \tag{18}$$

and

$$\mathbf{F}_2 = (\mathbf{F} - \mathbf{I})^{-1} = \mathbf{B}^{-1} \tag{19}$$

McArdle's formulation may thus be characterized as follows:

(1) For convenience, order the manifest variables in the vector **m**, and the latent variables in the vector **l**. The path model is then tested as a COSAN model of order 2, in which

(2) $\mathbf{F}_1 = \left[\mathbf{I} \mid \mathbf{0}\right]$, where **I** is of order $p \times p$ and **0** is $p \times n$.

(3) $\mathbf{F}_2$ is the *inverse* of a square matrix $\mathbf{B}$ of "directed relationships." $\mathbf{B}$ is constructed from the path diagram as follows. Set all diagonal entries of $\mathbf{B}$ to $-1$. Examine the path diagram for arrows. For each arrow pointing from $v_j$ to $v_i$, record its path coefficient in position $b_{ij}$ of matrix $\mathbf{B}$.

(4) **P**, a symmetric matrix, contains coefficients for "undirected" paths between variables $v_j$ and $v_i$ recorded in positions $p_{ij}$ and $p_{ji}$.

## The Bentler-Weeks Model

The RAM model is somewhat wasteful in terms of the size of some of its matrices. Bentler and Weeks (1979) produced an alternative model which is somewhat more efficient in the size of its matrices. Specifically, the $\mathbf{F}_2$ and **P** matrices are quite large in the RAM model, and have a large number of zero elements. Bentler and Weeks showed how, in situations where there are no manifest exogenous variables (i.e., all manifest variables have at least one arrow pointing to them), the McArdle-McDonald approach may be modified to reduce the size of the model matrices.

Partition **v** in the form

$$\mathbf{v} = \begin{bmatrix} \mathbf{m}_n \\ \hline \mathbf{l}_n \\ \hline \mathbf{l}_x \end{bmatrix} \tag{20}$$

where **m** stands for "manifest," **l** for "latent," the subscripts $x$ and $n$ refer to "exogenous" and "endogenous," respectively.

Then one may write $\mathbf{v} = \mathbf{F}\mathbf{v} + \mathbf{r}$ in a partitioned form as

$$\mathbf{v} = \begin{bmatrix} \mathbf{m}_n \\ \hline \mathbf{l}_n \\ \hline \mathbf{l}_x \end{bmatrix} = \left[\begin{array}{c:c:c} \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 \\ \hdashline \mathbf{F}_4 & \mathbf{F}_5 & \mathbf{F}_6 \\ \hdashline \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right] \begin{bmatrix} \mathbf{m}_n \\ \hline \mathbf{l}_n \\ \hline \mathbf{l}_x \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \hline \mathbf{0} \\ \hline \mathbf{l}_x \end{bmatrix} \tag{21}$$

Now define $\mathbf{n}$ as a vector containing all the endogenous, or "dependent" variables. We may partition $\mathbf{m}$ into manifest exogenous and endogenous variables, i.e.,

$$\mathbf{m} = \begin{bmatrix} \mathbf{m}_x \\ \hline \mathbf{m}_n \end{bmatrix} \tag{22}$$

Then

$$\mathbf{n} = \begin{bmatrix} \mathbf{m}_n \\ \hline \mathbf{l}_n \end{bmatrix} \tag{23}$$

One may then write

$$\mathbf{n} = \mathbf{F}_0\mathbf{n} + \mathbf{\Gamma}\mathbf{l}_x \tag{24}$$

where

$$\mathbf{F}_0 = \left[\begin{array}{c:c} \mathbf{F}_1 & \mathbf{F}_2 \\ \hdashline \mathbf{F}_4 & \mathbf{F}_5 \end{array}\right], \text{and } \mathbf{\Gamma} = \begin{bmatrix} \mathbf{F}_3 \\ \hline \mathbf{F}_6 \end{bmatrix} \tag{25}$$

The derivation now proceeds with an algebraic development similar to the RAM-COSAN equations. Rearranging Equation 24, one obtains

$$(\mathbf{I} - \mathbf{F}_0)\mathbf{n} = \mathbf{\Gamma}\mathbf{l}_x \tag{26}$$

$$\mathbf{n} = (\mathbf{I} - \mathbf{F}_0)^{-1}\mathbf{\Gamma}\mathbf{l}_x \tag{27}$$

$$\mathbf{m}_n = \left[\begin{array}{c:c} \mathbf{I} & \mathbf{0} \end{array}\right]\mathbf{n} = \left[\begin{array}{c:c} \mathbf{I} & \mathbf{0} \end{array}\right](\mathbf{I} - \mathbf{F}_0)^{-1}\mathbf{\Gamma}\mathbf{l}_x = \mathbf{J}(\mathbf{I} - \mathbf{F}_0)^{-1}\mathbf{\Gamma}\mathbf{l}_x \tag{28}$$

whence, letting

$$\mathbf{G} = \left[\begin{array}{c:c} \mathbf{I} & \mathbf{0} \end{array}\right],\ \mathbf{F}_2 = (\mathbf{I} - \mathbf{F}_0)^{-1},\ \mathbf{F}_3 = \mathbf{\Gamma},\ \text{and } \mathbf{P} = E(\mathbf{l}_x\mathbf{l}_x{}') \tag{29}$$

we have

$$\mathbf{\Sigma} = \mathbf{G}\mathbf{F}_2\mathbf{F}_3\mathbf{P}\mathbf{F}_3{}'\mathbf{F}_2{}'\mathbf{G}' \tag{30}$$

$\mathbf{G}$ is a filter matrix similar to $\mathbf{J}$ in the McArdle-McDonald specification. $\mathbf{F}_2 = \mathbf{B}_2^{-1}$, where $\mathbf{B}_2$ is a matrix containing path coefficients for directed relationships *among endogenous variables only*, and having $-1$ as each diagonal element. $\mathbf{F}_3$ contains path coefficients *from exogenous variables to endogenous variables only*, and $\mathbf{P}$ contains coefficients for undirected relationships, i.e., the variance-covariance parameters for the latent exogenous variables.

This clever algebraic refinement allowed some of the virtues of the McArdle approach to be retained, while expressing the essential relationships in smaller matrices. (Notice how several of the null submatrices are eliminated.) However, this model also had some minor drawbacks. It required partitioning variables into exogenous and endogenous types, and it did not allow explicit expression of manifest exogenous variables.

An alternative model allows us to treat manifest exogenous variables explicitly. If you add a vector of manifest variables to each of the two variable lists in the Bentler-Weeks (1979) model, and modify the regression coefficient matrices accordingly, you arrive at the model used by Steiger (1994) in *SEPATH*. In this model, which is similar to one given by Bentler and Weeks (1980), variables are partitioned into two groups.

## The SEPATH Model

Partition all the variables in the path diagram into vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ as follows:

$$\mathbf{s}_1 = \begin{bmatrix} \mathbf{m}_x \\ \hline \mathbf{m}_n \\ \hline \mathbf{l}_n \end{bmatrix} \tag{31}$$

and

$$\mathbf{s}_2 = \begin{bmatrix} \mathbf{m}_x \\ \hline \mathbf{l}_x \end{bmatrix} \tag{32}$$

Then one may write

$$\mathbf{s}_1 = \mathbf{B}\mathbf{s}_1 + \mathbf{\Gamma}\mathbf{s}_2 \tag{33}$$

where

$$\mathbf{B} = \left[\begin{array}{c|c|c} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{F}_1 & \mathbf{F}_2 \\ \hline \mathbf{0} & \mathbf{F}_4 & \mathbf{F}_5 \end{array}\right] \tag{34}$$

and

$$\mathbf{\Gamma} = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{F}_7 & \mathbf{F}_3 \\ \hline \mathbf{F}_8 & \mathbf{F}_6 \end{array}\right] \tag{35}$$

Assuming a nonsingular $\mathbf{I} - \mathbf{B}$, Equation 33 may be rewritten as

$$\mathbf{s}_1 = (\mathbf{I} - \mathbf{B})^{-1}\mathbf{\Gamma}\mathbf{s}_2 \tag{36}$$

Let $\mathbf{B}$ be a filter matrix which extracts the manifest variables from $\mathbf{s}_1$, and let $\mathbf{\Xi} = E(\mathbf{s}_2\mathbf{s}_2{}')$ be the covariance matrix for $\mathbf{s}_2$.

Then

$$\mathbf{m} = \left[\begin{array}{c} \mathbf{m}_x \\ \hline \mathbf{m}_n \end{array}\right] = \mathbf{G}\mathbf{s}_1 = \mathbf{G}(\mathbf{I} - \mathbf{B})^{-1}\mathbf{G}\mathbf{s}_2 \tag{37}$$

and one obtains the following model for covariance structure:

$$\mathbf{\Sigma} = \mathbf{G}(\mathbf{B} - \mathbf{I})^{-1}\mathbf{\Gamma}\mathbf{\Xi}\mathbf{\Gamma}'(\mathbf{B}' - \mathbf{I})^{-1}\mathbf{G}' \tag{38}$$

The covariance matrix $\mathrm{Cov}(\mathbf{s}_1) = \mathbf{\Psi}$ for manifest exogenous, manifest endogenous, and latent endogenous variables may be computed as

$$\mathbf{\Psi} = (\mathbf{B} - \mathbf{I})^{-1}\mathbf{\Gamma}\mathbf{\Xi}\mathbf{\Gamma}'(\mathbf{B}' - \mathbf{I})^{-1} \tag{39}$$

The model of Equation 38 allows direct correspondence between all permissible PATH1 statements and the algebraic model. There is no need to concoct dummy latent variables. All possible types of relationships among manifest and latent variables are accounted for. After a model is complete, all variables can immediately be assigned to one of the 4 vectors $\mathbf{m}_n$, $\mathbf{m}_x$, $\mathbf{l}_n$, or $\mathbf{l}_x$. All coefficients (for arrows) are then assigned to the matrices $\mathbf{F}_1$ through $\mathbf{F}_8$. The column index for a variable (in any of these 8 matrices) represents the variable from which the arrow points, the row index the variable to which the arrow points. Coefficients for wires or two-headed arrows ("slings") are represented in a similar manner in the matrix $\mathbf{\Xi}$.

The model of Equation 38 sacrifices some of the simplicity of the RAM model, because variables must be assigned to 4 types before the location of model coefficients can be determined. However, in our typology and with the *SEPATH* diagramming rules the typing of each variable into one of 4 categories can be determined by looking *only at that variable in the path diagram.* Because two headed arrows are eliminated, a variable is endogenous if and only if it has an arrowhead directed toward it. A variable is latent if and only if it appears in an oval or circle. (If it is not already obvious, let us note that with two headed arrows one must look away from the variable of interest to determine if the variable is endogenous, because an arrowhead attached to the variable and pointing to it might be two-headed! *Not only is the SEPATH system less cluttered, but it is also visually more efficient.)*

Two final points should be emphasized. First, it is not clear which of the above models is, in any overall sense, "superior" to the others. Second, it is possible to express some of the models as special cases of the others. For example, the LISREL model can be written easily as a COSAN model. To see why, suppose that the manifest and latent variables were ordered in the $\mathbf{v}$ of Equation 10 so that

$$\mathbf{v} = \begin{bmatrix} \mathbf{y} \\ \hline \mathbf{x} \\ \hline \boldsymbol{\eta} \\ \hline \boldsymbol{\xi} \end{bmatrix} \tag{40}$$

Then it follows immediately that one may write $\mathbf{v} = \mathbf{F}^{*}\mathbf{v} + \mathbf{r}^{*}$, where

$$\mathbf{F}^{*} = \left[\begin{array}{c:c:c:c} \mathbf{0} & \mathbf{0} & \boldsymbol{\Lambda}_y & \mathbf{0} \\ \hdashline \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\Lambda}_x \\ \hdashline \mathbf{0} & \mathbf{0} & \mathbf{B} & \boldsymbol{\Gamma} \\ \hdashline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right] \tag{41}$$

and

$$\mathbf{r}^* = \begin{bmatrix} \mathbf{e} \\ - \\ \mathbf{d} \\ - \\ \mathbf{z} \\ - \\ \mathbf{x} \end{bmatrix} \qquad (1)$$

If $\mathbf{P}^*$ is defined as the covariance matrix of $\mathbf{r}^*$, then clearly one can test any LISREL model as a COSAN model of the form

$$\mathbf{S} = \mathbf{G}\,(\mathbf{F}^* - \mathbf{I})^{-1}\,\mathbf{P}\,(\mathbf{F}'^* - \mathbf{I})^{-1}\mathbf{G}' \qquad (2)$$

where **G** is a matrix which filters **x** and **y** from **v**.