Homework 03

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Maximum Score = 100 points

1. (35 points). Murnane and Willett (2011), in their textbook *Methods Matter*, have a simplified, conceptual introduction to notions of power calculation, sample size estimation, and effect size in their Chapter 6. The chapter is available for downloading from the course website. The authors give a technical example which is in full development by page 87, on which there is a footnote 5 that purports to give a formula for the mean of the noncentral t distribution when the null hypothesis is false. This example concerns the two-sample, independent sample (between subjects) design described in detail in their Chapter 4. In this study, some randomly selected subjects received a school voucher (V) and some did not (NV). The null hypothesis is that

$$H_0: \quad \mu_V - \mu_{NV} = 0$$

The authors define

$$\delta = \mu_V - \mu_{NV}$$

then state in the footnote that the noncentral t has a mean of

$$\delta\sqrt{\nu/2}\frac{\Gamma((\nu-1)/2)}{\Gamma(\nu/2)}$$

with degrees of freedom $\nu = n_1 + n_2 - 2$. Read the chapter, and then answer the following questions.

- a. (5 points). The formula in footnote 5 has typos (mismatched parentheses, which I fixed in the version above), but is also obviously incorrect. Why? Something (a well-known statistical quantity expressed in a single Greek letter) is not present in the formula which any student with a proper general conception of power analysis should realize is missing. What is this missing quantity that is a "dead giveaway" that the formula is incorrect?
- b. (20 points). Suppose that the actual population statistics match the sample statistics, i.e., $\mu_V - \mu_{NV} = 4.899$, and $\sigma = 19.209$. Following the system we used in class, perform a simulation experiment with $n_1 = n_2 = 500$. Perform 100000 replications of a 2-sample t, and report the mean of the resulting statistics. Compare it to the mean calculated from the Murnane-Willett formula. To help you, I include R code to calculate the mean according to Murnane and Willett. Note how I use the `lgamma` function rather than the `gamma` function. The former gives the logarithm of the gamma. Note how I compute the ratio of two gammas by exponentiating the difference between two lgammas. This is to avoid overflow, because $\Gamma(n) = (n-1)!$ and becomes very large very quickly. Show your code. You will find that the simulation results fail to match the Murnane-Willett formula.
- c. (10 points). Go to Wikipedia and look up the mean (expected value) of a noncentral t variable. Compare it to results in my class notes on *The t distribution and its application* (slide 27) and the formula in Murnane and Willett. The exact nature of the error in Murnane and Willett (2011) should now be clear. Rewrite the Murnane-Willett formula so that it is correct for the two-sample t. Write an R formula to compute the correct mean, and, using it, show that your simulation results from part (b) match the corrected formula very closely.

2. (10 points). You wish to perform a 2-sample, independent sample t-test of the hypothesis

$$\mu_1 = \mu_2$$

in a situation in which $\alpha = 0.05$, and the standardized effect size

$$E_s = \frac{\mu_1 - \mu_2}{\sigma}$$

is believed to be 0.50. Assuming equal n per group, how large a sample size *per group* would you need to guarantee power of at least 0.90?

3. (25 points). In the same situation described in the previous problem, suppose the the standardized effect size E_s is to be estimated from sample data, based on sample sizes $n_1 = 20$, and $n_2 = 30$. You gather your data, and obtain a t statistic value of 2.922.

- a. (5 points). What are the degrees of freedom?
- b. (10 points). Give a 95% confidence interval for δ, the noncentrality parameter of the noncentral t statistic's distribution.
- c. (10 points). Convert this into a 95% confidence interval on E_s, the population standardized effect size (or "standardized mean difference").

4. (30 points). You are testing the hypothesis that $\mu_1 - \mu_2 - \mu_3 + \mu_4 = 0$ with 4 equal *independent* samples each of size $n = 25$. You observe a t statistic of 2.098.

- a. (5 points). What are the degrees of freedom?
- b. (10 points). Give a 95% confidence interval for δ, the noncentrality parameter of the noncentral t statistic's distribution.
- c. (15 points). Convert this into a 95% confidence interval on E_s, the population standardized effect size (or "standardized mean difference"). E_s is defined in this case as

$$E_s = \frac{\mu_1 - \mu_2 - \mu_3 + \mu_4}{\sigma}$$