Power Calculation in the 1-Way Random Effects ANOVA
Psychology 311
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The actual *F* statistic in 1-Way Random-Effects ANOVA is calculated in exactly the same way as the statistic for fixed effects. This might suggest that there is no substantive difference between the two designs. However, when the null hypothesis is false, the *F* statistic has a different distribution in the two designs.

In the fixed-effects design, the general distribution of the *F* statistic, as we saw previously, is noncentral *F*. In the random effects design, the formula below gives the distribution of the *F* statistic. To avoid some confusing notational clashes, I'll refer to the test statistic as *X*.

$$X \sim \left(1+\frac{n\sigma_A^2}{\sigma_e^2}\right) F_{A-1,A(n-1)}$$

Note that this is a constant times an *F* distribution. We can rewrite this as

$$X \sim CF_{A-1,A(n-1)}$$

where the "factor" *C* is

$$C = 1 + nV$$

and the "variance ratio" *V* is

$$V = \frac{\sigma_A^2}{\sigma_e^2}$$

Suppose we compute a rejection point F_{crit}. Power is simply

$$Power = \Pr(X > F_{crit}) = \Pr(CF_{A-1,A(n-1)} > F_{crit}) = \Pr\left(F_{A-1,A(n-1)} > \frac{F_{crit}}{C}\right)$$

So power is the probability of exceeding a "revised" critical value equal to the original critical value divided by *C*. Obviously, if $C = 1$, the revised critical value is equal to the original one and power is equal to α.

It is convenient to calculate power with respect to *A*, *n*, α, and the variance ratio *V*, or, alternatively, in terms of the RMSSE, which, for this model is

$$RMSSE = \frac{\sigma_A}{\sigma_e} = \sqrt{V}$$

Note that, of course, $V = RMSSE^2$.

Here is R code to compute power. Remember, *n* is the sample size *per group*.

```
Power.OneWay.Random <- function(n,A,alpha,var.ratio){
 df.1 <- A-1
 df.2 <- A*(n-1)
 F.crit <- qf(1-alpha,df.1,df.2)
 C <- 1 + n * var.ratio
 pow <- 1-pf(F.crit/C,df.1,df.2)
 return(pow)
}
```

Let's do a couple of calculations.

First, what is power if the RMSSE is 0.50, and n = 10 in a 3 group ANOVA with $\alpha = 0.05$?

Our routine requires the variance ratio, which is the square of the RMSSE.

```
> Power.OneWay.Random(10,3,.05,.5^2)
[1] 0.3961995
```

Next, what is power if the variance ratio is 0.16 and n = *50* in a 6 group design?

```
> Power.OneWay.Random(50,6,.05,.16)
[1] 0.9399165
```