# One-Way ANOVA Calculations: In-Class Exercise

Psychology 311

Spring, 2013

1. You are planning an experiment that will involve 4 equally sized groups, including 3 experimental groups and a control. Each group will contain $n$ observations. Your expectation is that each of the 3 experimental treatments will have approximately the same effect, and that this effect will be small — roughly one-third a standard deviation improved performance over the control.

   (a) Calculate the 4 effects $\alpha_j = \mu_j - \mu$. Note that you will only be able to express them in standard deviation units. (*Hint.* At first this may seem impossible, but recall that if the effects must sum to zero, the knowledge that 3 group means all differ from the control by $\sigma/3$ allows you to define the 4th effect so that the 4 effects sum to zero.)

   (b) Once you have the $\alpha_j$ values, you should immediately be able to specify the *standardized* effect values $\alpha_j^* = \alpha_j/\sigma$. What are they?

   (c) Suppose that $n = 10$ per group. Recall from lecture that the $F$ statistic for testing the null hypothesis of equal group means has a general distribution that is noncentral $F$ with $a - 1$ and $a(n - 1)$ degrees of freedom and a noncentrality parameter $\lambda$ given by

   $$\lambda = n \sum_{j=1}^{a} \alpha_j^{*2}$$

   What is the value of $\lambda$ in this case?

   (d) If the $F$ test is to be conducted with $\alpha = 0.05$, what is the critical value (i.e., rejection point)? When calculating the rejection point $H_0$ is true and $\lambda = 0$.

   (e) What is the power of the test with the proposed value of $n = 10$? Use R to perform the calculation, then verify it with Gpower 3.

   (f) How large an $n$ would you need to obtain a power of .90?

2. Suppose that you run the above experiment, and obtain the data shown below.

| | Control | Exp1 | Exp2 | Exp3 |
|---|---|---|---|---|
| 1 | 118 | 107 | 133 | 134 |
| 2 | 121 | 165 | 154 | 176 |
| 3 | 97 | 121 | 91 | 171 |
| 4 | 86 | 126 | 63 | 159 |
| 5 | 118 | 87 | 62 | 118 |
| 6 | 45 | 135 | 164 | 125 |
| 7 | 119 | 83 | 96 | 100 |
| 8 | 92 | 100 | 129 | 60 |
| 9 | 91 | 144 | 128 | 163 |
| 10 | 72 | 119 | 105 | 111 |

(a) Perform a 1-way ANOVA on the data.

(b) Given the result, compute a 95% confidence interval on the non-centrality parameter $\lambda$. *Hint.* You can use the MBESS routine `conf.limits.ncf` or my noncentral distribution calculator *NDC.*

(c) Cohen's $f$ can be written as

$$f = \sqrt{\frac{\sum_{j=1}^{a}(\alpha_j/\sigma)^2}{a}}$$

Examine the formula for $\lambda$, and note that $f$ can be written as a monotonic, strictly increasing function of $\lambda$. Derive a formula for converting $\lambda$ to $f$, and use it to compute a 95% confidence interval on $f$.

# Key to One-Way ANOVA Calculations: In-Class Exercise

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1. You are planning an experiment that will involve 4 equally sized groups, including 3 experimental groups and a control. Each group will contain $n$ observations. Your expectation is that each of the 3 experimental treatments will have approximately the same effect, and that this effect will be small — roughly one-third a standard deviation improved performance over the control.

   (a) Calculate the 4 effects $\alpha_j = \mu_j - \mu$. Note that you will only be able to express them in standard deviation units. (*Hint*. At first this may seem impossible, but recall that if the effects must sum to zero, the knowledge that 3 group means all differ from the control by $\sigma/3$ allows you to define the 4th effect so that the 4 effects sum to zero.)

   *Answer*. The group means can be written in terms of their relationship to each other as $0, \sigma/3, \sigma/3, \sigma/3$. To express them as effects $\alpha_j$, you have to transform them by subtracting a constant so that their mean is zero. Since the current mean is $\sigma/4$, we need to subtract $\sigma/4$ from each one. The effects then become $-\sigma/4, \sigma/12, \sigma/12, \sigma/12$.

   (b) Once you have the $\alpha_j$ values, you should immediately be able to specify the *standardized* effect values $\alpha_j^* = \alpha_j/\sigma$. What are they?

   *Answer*. We simply divide them all by $\sigma$, obtaining $-1/4, 1/12, 1/12, 1/12$.

   (c) Suppose that $n = 10$ per group. Recall from lecture that the $F$ statistic for testing the null hypothesis of equal group means has a general distribution that is noncentral $F$ with $a - 1$ and $a(n - 1)$ degrees of freedom and a noncentrality parameter $\lambda$ given by

   $$\lambda = n \sum_{j=1}^{a} (\alpha_j/\sigma)^2 = n \sum_{j=1}^{a} \alpha_j^{*2}$$

   What is the value of $\lambda$ in this case?

   *Answer*. The value of $\lambda$ is

   $$10(1/16 + 1/144 + 1/144 + 1/144) = 10(12/144) = 5/6 = 0.8333$$

   (d) If the $F$ test is to be conducted with $\alpha = 0.05$, what is the critical value (i.e., rejection point)? When calculating the rejection point $H_0$ is true and $\lambda = 0$.

   *Answer*. The $F$ statistic has 3 and 36 degrees of freedom, and the critical value is

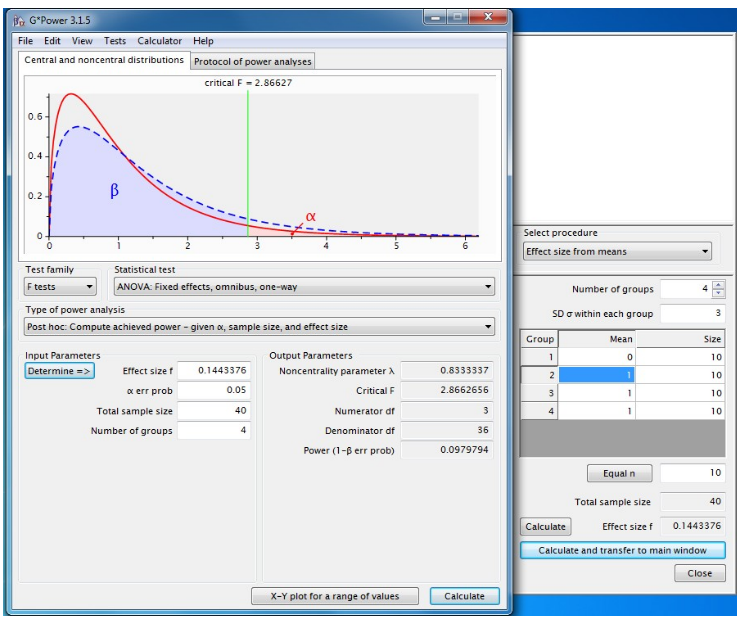
```
> F.crit <- qf(.95,3,36)
> F.crit
[1] 2.866266
```

(e) What is the power of the test with the proposed value of $n = 10$? Use R to perform the calculation, then verify it with Gpower 3.

*Answer.* Power is the area to the right of the rejection point we determined above in a noncentral $F$ distribution with 3 and 36 degrees of freedom and $\lambda = 0.8333$.

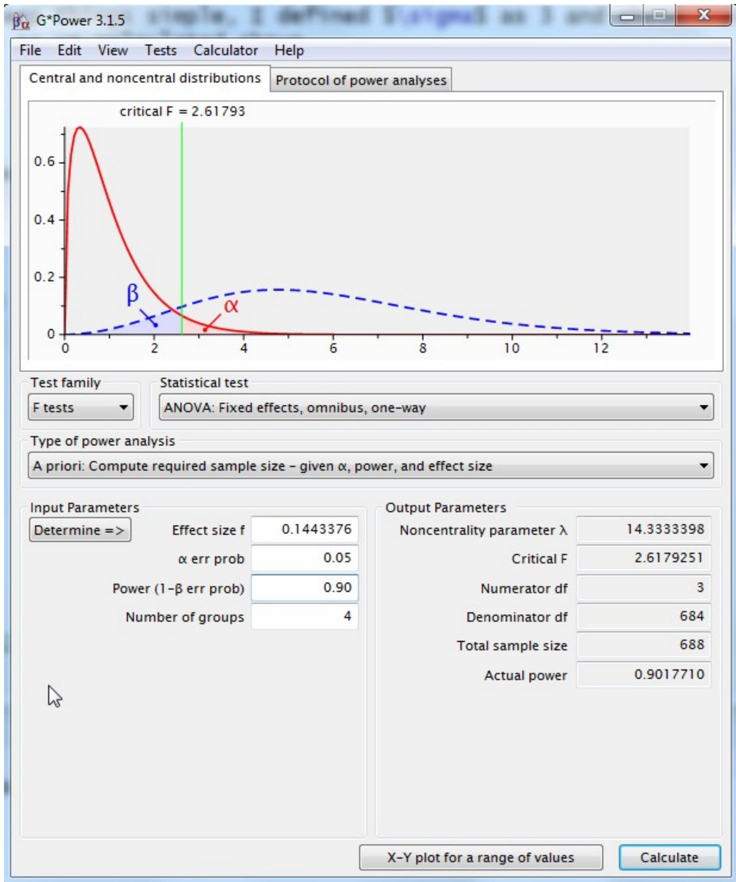
```
> lambda <- 5/6
> Power <- 1 - pf(F.crit,3,36,lambda)
> Power
[1] 0.09797938
```

Power is a pathetic 0.098. We can duplicate those calculations in Gpower 3 as shown in the following screen. Note how we open a side menu. To keep things simple, I defined $\sigma$ as 3 and chose the means to be 0,1,1,1 yielding the same standardized effects as we calculated above.




(f) How large an $n$ would you need to obtain a power of .90?

*Answer.* A total sample size of 688 for the 4 groups, or an $n$ of 172 per group, is required.


G*Power 3.1.5
File Edit View Tests Calculator Help
Central and noncentral distributions
Protocol of power analyses
critical F = 2.61793
0.6
0.4
0.2
0
β
α
0 2 4 6 8 10 12
Test family
F tests
Statistical test
ANOVA: Fixed effects, omnibus, one-way
Type of power analysis
A priori: Compute required sample size – given α, power, and effect size
Input Parameters
Determine =>
Effect size f 0.1443376
α err prob 0.05
Power (1-β err prob) 0.90
Number of groups 4
Output Parameters
Noncentrality parameter λ 14.3333398
Critical F 2.6179251
Numerator df 3
Denominator df 684
Total sample size 688
Actual power 0.9017710
X-Y plot for a range of values
Calculate

2. Suppose that you run the above experiment, and obtain the data shown below.

| | Control | Exp1 | Exp2 | Exp3 |
|---|---|---|---|---|
| 1 | 118 | 107 | 133 | 134 |
| 2 | 121 | 165 | 154 | 176 |
| 3 | 97 | 121 | 91 | 171 |
| 4 | 86 | 126 | 63 | 159 |
| 5 | 118 | 87 | 62 | 118 |
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| 10 | 72 | 119 | 105 | 111 |

(a) Perform a 1-way ANOVA on the data.

*Answer*. Typing in the data yields a data frame with 40 rows and 2 columns. The first and last few lines are shown below with the `head` and `tail` commands.

```
> head(data)
```

```
    Y   Group
1 118 Control
2 121 Control
3  97 Control
4  86 Control
5 118 Control
6  45 Control
```

```
> tail(data)
```

```
     Y Group
35 118  Exp3
36 125  Exp3
37 100  Exp3
38  60  Exp3
39 163  Exp3
40 111  Exp3
```

To analyze the data, one standard ANOVA approach is as follows.

```
> results <- anova(lm(Y ~ factor(Group)))
> xtable(results)
```

Note that, in the above code, I use the function `xtable` to produce typeset tables within LaTeX. The standard R output shown below would be produced simply by typing `results`.

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|---|---|---|---|---|---|
| factor(Group) | 3 | 6632.80 | 2210.93 | 2.31 | 0.0927 |
| Residuals | 36 | 34457.60 | 957.16 | | |

```
> results
Analysis of Variance Table

Response: Y
              Df Sum Sq Mean Sq F value  Pr(>F)
factor(Group)  3   6633 2210.93  2.3099 0.09274 .
Residuals     36  34458  957.16
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

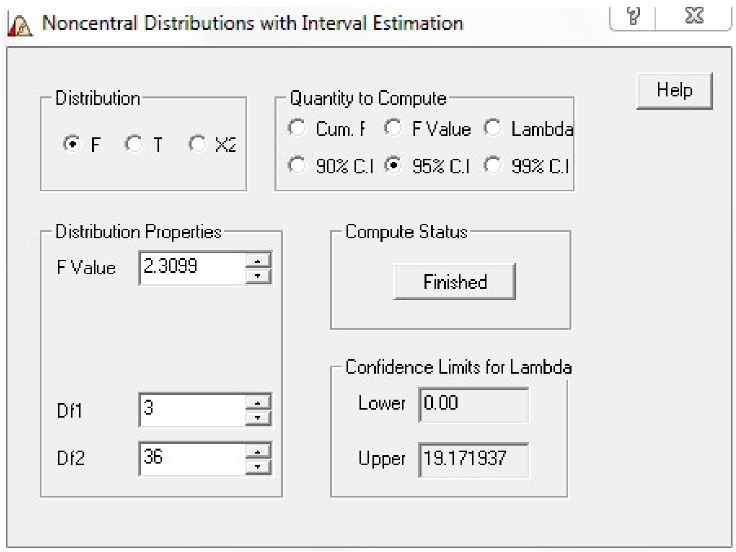
(b) Given the result, compute a 95% confidence interval on the noncentrality parameter $\lambda$. *Hint.* You can use the MBESS routine `conf.limits.ncf` or my noncentral distribution calculator *NDC*.

*Answer.* Here is MBESS output

```
> library(MBESS)
> lambda.ci <- conf.limits.ncf(F.value = 2.3099, df.1 = 3, df.2 = 36 )
> lambda.ci
$Lower.Limit
[1] NA

$Prob.Less.Lower
[1] NA

$Upper.Limit
[1] 19.17194

$Prob.Greater.Upper
[1] 0.025
```

Note that it returns a missing value code `NA` for the lower value. Since the lower limit cannot be lower than zero, I chose in my routine to return a value of zero.

Here is the NDC screen.




(c) Cohen's $f$ can be written as

$$f = \sqrt{\frac{\sum_{j=1}^{a}(\alpha_j/\sigma)^2}{a}}$$

Examine the formula for $\lambda$, and note that $f$ can be written as a monotonic, strictly increasing function of $\lambda$. Derive a formula for converting $\lambda$ to $f$, and use it to compute a 95% confidence interval on $f$.

*Answer.* We can write

$$f = \sqrt{\frac{\lambda}{na}}$$

```
> n <- 10; a <- 4
> upper  <- sqrt(lambda.ci$Upper.Limit/(n*a))
> upper
[1] 0.6923138
```

So our confidence interval for $f$ has limits of 0 and 0.6923.